Newtonian noise mitigation by using mini-SOGROs

Ho Jung Paik, 4/28/2016

Since SOGRO is a very sensitive gravity strain gauge, one may be able to employ scaled-down SOGROs, in place of a large array of seismometers, to *directly measure and remove* the Newtonian noise (NN) affecting the interferometer test masses. Here we investigate the possibility of mitigating the NN in interferometers by using mini-SOGROs with arm-length $\ell << L$.

Principle of NN mitigation

Rayleigh waves are expected to dominate the NN at $f \sim 10$ Hz [1]. We consider a Rayleigh wave traveling along a direction with an angle ψ with respect to the x axis. The displacements of a (free) test mass at height z above the ground along x, y, and z are given [2, 3] by

$$X(\omega) = -2\pi i \cos\psi\, G\rho_0 \gamma_R \frac{\xi(\psi,\omega)}{\omega^2} \exp\left[\frac{\omega}{c_R}(ix'-z)\right], \tag{1a}$$

$$Y(\omega) = -2\pi i \sin\psi\, G\rho_0 \gamma_R \frac{\xi(\psi,\omega)}{\omega^2} \exp\left[\frac{\omega}{c_R}(ix'-z)\right], \tag{1b}$$

$$Z(\omega) = -2\pi G\rho_0 \gamma_R \frac{\xi(\psi,\omega)}{\omega^2} \exp\left[\frac{\omega}{c_R}(ix'-z)\right], \tag{1c}$$

where $\xi(\psi,\omega)$ is the vertical displacement directly beneath the test mass, $\gamma_R \approx 0.83$ is a factor that accounts for partial cancellation for NN from surface displacement, $c_R \approx 250$ m/s and 3.4 km/s is the speed of waves at the ground level and underground, $x' \equiv x \cos\psi + y \sin\psi$ is the position of the test mass along the direction of wave propagation, and ρ_0 is the mean mass ground density.

In the presence of a GW and Rayleigh waves, the interferometer measures

$$\Delta L_x(\omega) - \Delta L_y(\omega) = hL + [X(\omega,x_2) - X(\omega,x_1)] - [Y(\omega,y_2) - Y(\omega,y_1)], \tag{2}$$

where $X(\omega, x_i)$ and $Y(\omega, y_i)$, $i = 1, 2$, represent the NN-induced displacements along the x and y axes, summed over multiple waves, at the test mass position x_i and y_i, respectively. At 10 Hz, the Rayleigh wavelength becomes $\lambda_R \sim 25$ m $<< L$, causing $X(\omega, x_i)$ and $Y(\omega, x_i)$ to be completely uncorrelated with one another. Hence we need to measure $X(\omega, x_i)$ or $Y(\omega, x_i)$ for each test mass by using a separate mini-SOGRO co-located with it, as shown in Fig. 1.

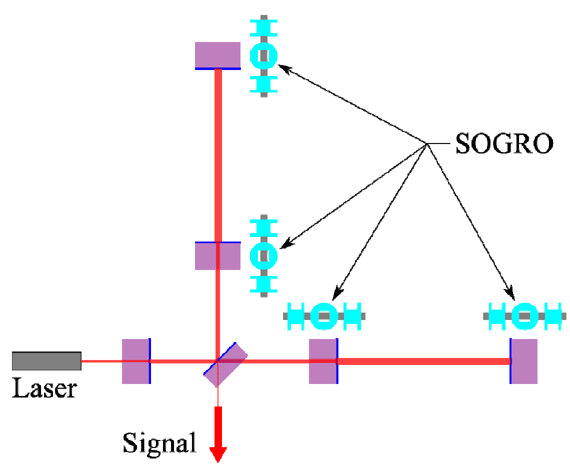

Fig. 1. Four mini-SOGROs collocated with four test masses of a laser interferometer.

The strain tensor that a mini-SOGRO detects can be shown [3] to be

$$h'_{ij}(\omega) = h_{ij}(\omega) + 2\eta \frac{\xi(\psi,\omega)}{\omega} \exp\left[\frac{\omega}{c_R}(ik - z)\right] \begin{pmatrix} \cos^2\psi & \cos\psi\sin\psi & -i\cos\psi \\ \cos\psi\sin\psi & \sin^2\psi & -i\sin\psi \\ -i\cos\psi & -i\sin\psi & -1 \end{pmatrix}, \tag{3}$$

$$\eta \equiv 2\pi G\rho_0 \frac{\gamma_R}{c_R}. \tag{4}$$

By comparing Eq. (3) with Eq. (1a), we find

$$h'_{13}(\omega) = -\frac{2i\omega}{c_R} X(\omega), \quad h'_{23}(\omega) = -\frac{2i\omega}{c_R} Y(\omega), \tag{5}$$

where $h_{13}(\omega)$ was ignored since it is completely dominated by the NN. Therefore, the NN in the interferometer test mass along the x (y) axis could *in principle* be mitigated by co-locating a mini-SOGRO with it (see Fig. 1), and correlating the 13- (23-) component of the mini-SOGRO with the interferometer output and subtracting the correlated part.

We solve Eq. (5) for $X(\omega)$ and substitute it into Eq. (2) to obtain

$$h(\omega) = \frac{1}{L}\left[\Delta L_x(\omega) - \Delta L_y(\omega)\right] + \frac{ic_R}{2\omega L}\left[h'_{13}(\omega, x_2) - h'_{13}(\omega, x_1)\right] - \frac{ic_R}{2\omega L}\left[h'_{23}(\omega, y_2) - h'_{23}(\omega, y_1)\right]. \tag{6}$$

The sensitivity required for mini-SOGRO to recover h is then given by

$$h' = \frac{\omega L}{c_R} h, \tag{7}$$

where the numerical factor came from the incoherent sum of the noise in the four mini-SOGROs. Figure 2 shows the sensitivity goals of Advanced LIGO (aLIGO) and Einstein Telescope (ET) [1]. The shaded region represents the parameter space dominated by the NN. A worthy mitigation goal for aLIGO would be rejecting the NN by a factor of 5 to 2×10^{-23} Hz$^{-1/2}$ at 10 Hz. With $L = 4$ km, Eq. (7) yields $h' = 2 \times 10^{-20}$ Hz$^{-1/2}$ at 10 Hz.

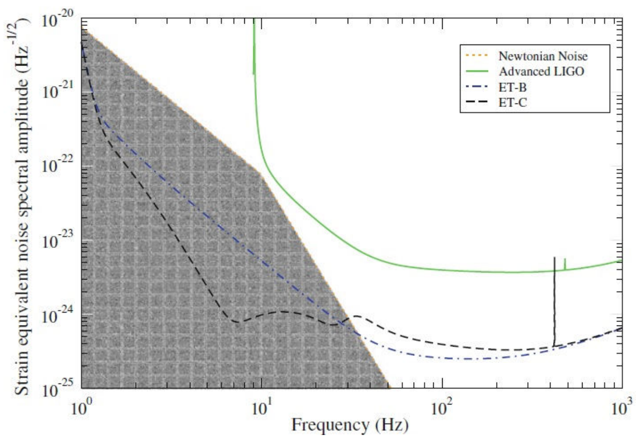

Fig. 2. Sensitivity goals of aLIGO and ET. The shaded region is dominated by the NN.

To be able to mitigate the NN by using this correlation method, the SOGRO output must be highly correlated with the NN-induced displacement of the test mass. The mitigation factor S is related to the correlation between the sensor and the test mass, C_{SN}, [1] as

$$S = \frac{1}{\sqrt{1 - C_{SN}^2}}. \tag{8}$$

To achieve $S = 5$, one needs $C_{SN} = 0.98$. This requires $\ell \leq 0.8$ m and each SOGRO must be located within 0.8 m of the interferometer test mass [4]. Such a small SOGRO would hardly have enough sensitivity and could not be brought to such proximity to the interferometer test mass.

Two mini-SOGROs symmetrically located

We consider the possibility of locating *two* larger baseline mini-SOGROs symmetrically on the opposite sides of each test mass and *averaging out the uncorrelated parts*. Figure 3 shows the four test masses of the two mini-SOGROs on the horizontal plane at the same z as the interferometer test mass M. For simplicity, the four vertical test masses are not shown.

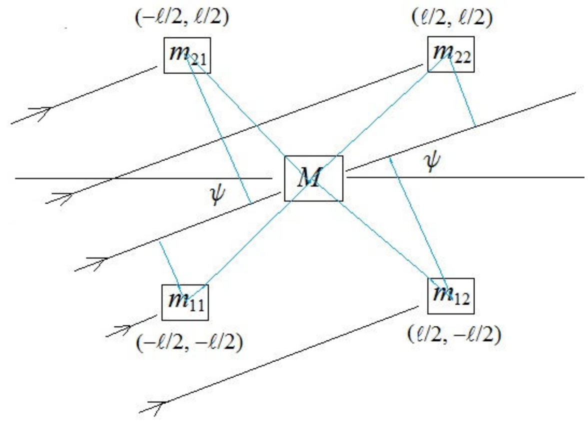

Fig. 3. Four mini-SOGRO test masses surrounding an interferometer test mass.

We choose the origin to be at M. Then, the NN-driven displacement of M along the x axis is

$$X(\omega) = -i\eta c_R \cos\psi \frac{\xi(\psi,\omega)}{\omega^2} e^{-kz}, \tag{9}$$

where $k \equiv \omega/c_R$. The 13-components of the mini-SOGRO 1 and 2 response are given by

$$h_{13}^{(1)}(\omega) = \frac{1}{\ell}\{[Z(x'_{12},\omega) - Z(x'_{11},\omega)] + [X(z_{14},\omega) - X(z_{13},\omega)]\}, \tag{10a}$$

$$h_{13}^{(2)}(\omega) = \frac{1}{\ell}\{[Z(x'_{22},\omega) - Z(x'_{21},\omega)] + [X(z_{24},\omega) - X(z_{23},\omega)]\}, \tag{10b}$$

where x'_{ij} and z_{ij} are the x' and z positions of test mass ij. From Fig. 3, we find

$$x'_{11} = -x'_{22} = -\frac{\ell}{\sqrt{2}}\cos\left(\frac{\pi}{4} - \psi\right),\ x'_{12} = -x'_{21} = \frac{\ell}{\sqrt{2}}\sin\left(\frac{\pi}{4} - \psi\right), \tag{11a}$$

$$z_{13} = z_{23} = z - \frac{\ell}{2},\ z_{14} = z_{24} = z + \frac{\ell}{2}. \tag{11b}$$

Substituting Eqs. (11) and (1) into Eqs. (10), we obtain the response of mini-SOGRO 1 and 2:

$$h_{13}^{(1)}(\omega) = \frac{\eta c_R}{\ell}\frac{\xi(\psi,\omega)}{\omega^2} e^{-kz} \left\{ \begin{aligned} & -\exp\left[i\frac{k\ell}{\sqrt{2}}\sin\left(\frac{\pi}{4}-\psi\right)\right] + \exp\left[-i\frac{k\ell}{\sqrt{2}}\cos\left(\frac{\pi}{4}-\psi\right)\right] \\ & + i\cos\psi \exp\left[i\frac{k\ell}{2\sqrt{2}}\sin\left(\frac{\pi}{4}-\psi\right) - i\frac{k\ell}{2\sqrt{2}}\cos\left(\frac{\pi}{4}-\psi\right)\right]\left[\exp\left(\frac{k\ell}{2}\right) - \exp\left(-\frac{k\ell}{2}\right)\right] \end{aligned} \right\}, \tag{12a}$$

$$h_{13}^{(2)}(\omega) = \frac{\eta c_R}{\ell}\frac{\xi(\psi,\omega)}{\omega^2} e^{-kz} \left\{ \begin{aligned} & -\exp\left[i\frac{k\ell}{\sqrt{2}}\cos\left(\frac{\pi}{4}-\psi\right)\right] + \exp\left[-i\frac{k\ell}{\sqrt{2}}\sin\left(\frac{\pi}{4}-\psi\right)\right] \\ & + i\cos\psi \exp\left[i\frac{k\ell}{2\sqrt{2}}\cos\left(\frac{\pi}{4}-\psi\right) - i\frac{k\ell}{2\sqrt{2}}\sin\left(\frac{\pi}{4}-\psi\right)\right]\left[\exp\left(\frac{k\ell}{2}\right) - \exp\left(-\frac{k\ell}{2}\right)\right] \end{aligned} \right\}. \tag{12b}$$

The sum over the two mini-SOGROs becomes

$$Y(\omega) \equiv h_{13}^{(1)}(\omega) + h_{13}^{(2)}(\omega) = \frac{2i\eta c_R}{\ell}\cos\psi \frac{\xi(\psi,\omega)}{\omega^2} e^{-kz} \cos\left(\frac{k\ell}{2}\sin\psi\right)\left[\exp\left(\frac{k\ell}{2}\right) - \exp\left(-\frac{k\ell}{2}\right)\right]. \tag{13}$$

Surprisingly, the horizontal axis terms cancel out and only the vertical axis terms contribute.

Now, we superpose Rayleigh waves coming from all directions, $\psi \in (0, 2\pi)$. Equations (9) and (13) are modified into

$$X(\omega) = -\frac{i\eta c_R}{\omega^2} e^{-kz} \int_0^{2\pi} d\psi \xi(\psi, \omega) \cos\psi, \tag{14}$$

$$Y(\omega) = \frac{2i\eta c_R}{\ell \omega^2} e^{-kz} \left[\exp\left(\frac{k\ell}{2}\right) - \exp\left(-\frac{k\ell}{2}\right) \right] \int_0^{2\pi} d\psi \xi(\psi, \omega) \cos\psi \cos\left(\frac{k\ell}{2} \sin\psi\right). \tag{15}$$

In the limit $k\ell/2 << 1$ (small ℓ),

$$Y(\omega) \to \frac{2i\eta}{\omega} e^{-kz} \int_0^{2\pi} d\psi \xi(\psi, \omega) \cos\psi \tag{16}$$

and the correlation between $X(\omega)$ and $Y(\omega)$ approaches unity, as expected.

The correlation between the double mini-SOGRO and interferometer test mass is given by

$$C_{SN} = \frac{\int_0^{2\pi} d\psi X(\psi) Y^*(\psi)}{\sqrt{\int_0^{2\pi} d\psi X(\psi) X^*(\psi) \int_0^{2\pi} d\psi Y(\psi) Y^*(\psi)}}, \tag{17}$$

where

$$X(\psi) = -\frac{i\eta c_R}{\omega^2} e^{-kz} \xi(\psi, \omega) \cos\psi, \tag{18}$$

$$Y(\psi) = \frac{2i\eta c_R}{\ell \omega^2} e^{-kz} \left[\exp\left(\frac{k\ell}{2}\right) - \exp\left(-\frac{k\ell}{2}\right) \right] \xi(\psi, \omega) \cos\psi \cos\left(\frac{k\ell}{2} \sin\psi\right). \tag{19}$$

Substituting Eqs. (18) and (19) into Eq. (17), we obtain

$$C_{SN} = \frac{\int_0^{2\pi} d\psi \left|\xi(\psi, \omega)\right|^2 \cos^2\psi \cos\left(\frac{k\ell}{2} \sin\psi\right)}{\sqrt{\int_0^{2\pi} d\psi \left|\xi(\psi, \omega)\right|^2 \cos^2\psi \int_0^{2\pi} d\psi \left|\xi(\psi, \omega)\right|^2 \cos^2\psi \cos^2\left(\sin\psi \frac{k\ell}{2}\right)}}, \tag{20}$$

where $\xi(\psi,\omega)$ is a function of ψ with random amplitude and phase. Equation (20) shows that the phase does not contribute to C_{SN}.

Figure 4 shows C_{SN} computed as a function of $k\ell$. In order to obtain $C_{SN} = 0.98$, we need $k\ell \le 2.4$. At $f = 10$ Hz, $k = 0.25$ m^{-1} and $\ell \le 9.5$ m. Therefore, the uncorrelated parts of the NN could indeed be averaged out by symmetrically located mini-SOGROs with a larger baseline (9.5 m instead of 0.8 m for $S = 5$).

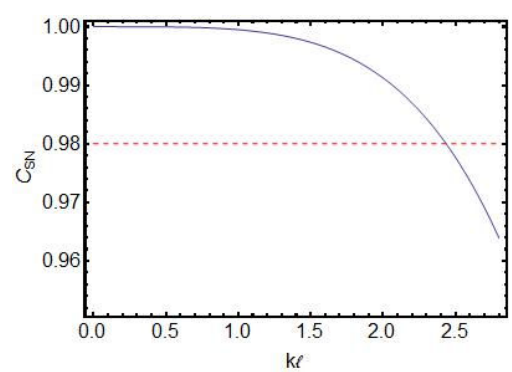

Fig. 4. Correlation between the double mini-SOGRO and interferometer test mass versus $k\ell$.

Table 1. Proposed detector parameters and detector noise of mini-SOGRO.

Parameter	SOGRO	Method Employed
Each test mass M	1.5×10^3 kg	Nb square tube
Arm-length L	4 m	Over a rigid platform
Antenna temperature T	4.2 K	Liquid helium or cryocooler
SQUID temperature T_{SQ}	0.1 K	He^3/He^4 dilution refrigerator
DM resonance frequency f_D	10^{-2} Hz	Magnetic levitation
DM quality factor Q_D	10^7	Surface polished pure Nb
Pump frequency f_p	50 kHz	Tuned capacitor bridge transducer
Amplifier noise number n	10	Nearly quantum-limited dc SQUID
Detector noise $S_h^{1/2}(f)$	2×10^{-20} Hz$^{-1/2}$	Computed at $f = 10$ Hz

Table 1 shows detector parameters that could meet the SOGRO sensitivity requirement. Each test mass weighs 1.5 tons and the baseline is 4 m. It would be sufficient to cool the test masses to 4.2 K, as long as the SQUIDs are cooled separately to 0.1 K to reach the required noise level of $10\hbar$. A white noise level of $10\hbar$ at 0.1 K has been demonstrated by using a two-stage dc SQUID [5]. The Q requirement for the test masses is modest.

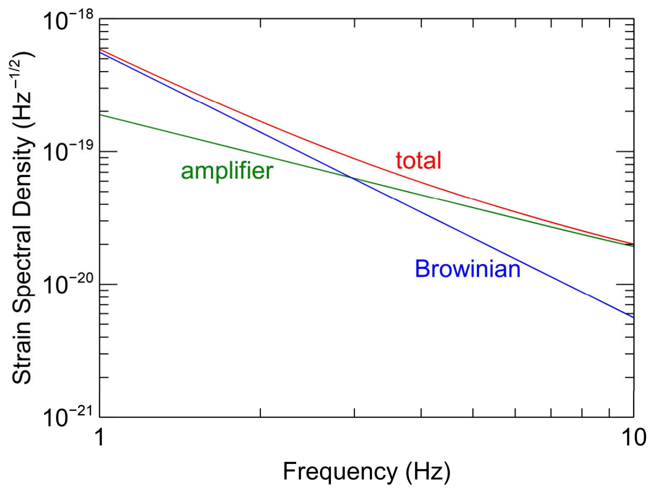

Fig. 5. Detector noise of a 4-m SOGRO cooled to 4.2 K and coupled to a $10\hbar$ SQUID.

The intrinsic noise spectral density computed for these parameters is plotted in Fig. 5. The total detector noise at 10 Hz is 2×10^{-20} Hz$^{-1/2}$, satisfying our requirement for $S = 5$.

A single mini-SOGRO located under each test mass

We could avoid the complexity of using two mini-SOGROs for each interferometer test mass, if a mini-SOGRO could precisely be co-located with each test mass. We could further simplify it if we could dispense with the two test masses that measure the relative vertical acceleration induced by h'_{13} and h'_{23}.

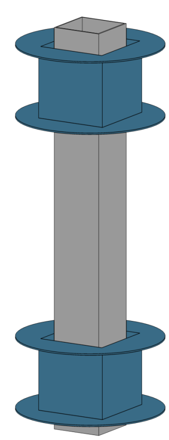

Fig. 6. A single-arm mini-SOGRO located under each test mass.

Due to nonlinearities in the superconducting circuit, vertical resonance frequencies of levitated superconducting test masses remain high ($\geq$ 1 Hz). In principle, the superconducting negative spring that has been demonstrated with SGG [6] could be applied but the actual implementation would be very challenging. Normally, the angular response of the two test masses on the horizontal arm and that of the two test masses on the vertical arm are differenced to reject the CM angular acceleration of the platform. If the platform could be sufficiently well isolated from the seismic noise of the ground, one would not need such differencing. Figure 6 shows a single-arm mini-SOGRO located under each interferometer test mass.

The platform could be suspended as a pendulum from its mid-point. Under such suspension, the platform would be completely isolated from the ground tilt. By keeping its angular resonance frequency about the *z* axis to 1 mHz, the angular acceleration about the *z* axis would be isolated by 10^8 at 10 Hz. A typical seismic noise level at shallow depth (~10 m) is ~10^{-7} m s^{-2} $Hz^{-1/2}$ [7]. To reach the intrinsic noise level of 2×10^{-20} $Hz^{-1/2}$, the linear acceleration noise must be rejected by 10^{10} at 10 Hz. The mini-SOGRO could be designed to have a total CM linear acceleration rejection ratio of 10^{10}.

There is concern that the underground cavity which houses the mini-SOGRO may cause Rayleigh waves to scatter in a way that produces NN signals for the SOGRO test masses that are not completely correlated with the NN that affects the interferometer test mass [4]. This problem needs to be studied.

References

[1] M. G. Beker *et al*., "Improving the sensitivity of future gravitational wave observatories in the 1–10 Hz band: Newtonian and seismic noise," *Gen. Rel. Grav*. **43**, 623 (2011).

[2] J. Harms *et al.,* "Low frequency terrestrial gravitational wave detectors," *Phys. Rev. D* **88** 122003 (2013).

[3] H. J. Paik, "Mitigation of Newtonian noise using a tensor gravitational wave detector," unpublished notes (2016).

[4] J. Harms, private communications.

[5] P. Falferi *et al*., "10$\hbar$ SQUID amplifier for acoustic gravitational wave detectors," *Appl. Phys. Lett*. **93** 172506 (2008).

[6] J. W. Parke, H. J. Paik, H. A. Chan and M. V. Moody, "Sensitivity Enhancement of Inertial Instruments by Means of a Superconducting Negative Spring," in the *Proceedings of the 10th International Cryogenic Engineering Conference*, edited by H. Collan, P. Berglund, and M. Krusius (Butterworth, Surrey, 1984), p.361.

[7] M. G. Beker *et al*., "Newtonian noise and ambient ground motion for gravitational wave detectors," *J. Phys. Conf. Ser*. **363**, 012004 (2012).